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IRREDUNDANT GIRTH 2-DOMINATION NUMBER OF CORONA GRAPHS AND ITS MATCHINGS

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ABSTRACT

A range set D in G∘ H is a girth 2-dominating set of G∘ H if every vertex in V-D is adjacent to atleast two vertex of girth graph is called the Irredundant girth 2-dominating set . the matching number is the maximum cardinality of a matching of G, while the irredundant girth 2-domination number of G∘ H is the minimum cardinality taken over all irredundant girth 2-domination number and is denoted by $\gamma_{m2irg}(G \circ H)$.

KEYWORDS:

Matching ;

Domination Number;

Girth 2 Domination ; Irredundant Girth 2 Domination Number ;

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1.INTRODUCTION:

Consider a circle where each range of set of domain is changed from inner portion to a image or co-domain in the outer vertices . We call this circle as 'cipher circle'. In this way if we try to change the vertices. Now we use the same circle if we substitute for each outer vertices corresponding to the inner vertices. The range function we get it is a bijective mapping of relation which is called bijective range function of Corona Graph. For a given vertex v of a graph G , The open neighbourhood of v in G is the set $N_G(v)$ of all vertices of G that are adjacent to v. The degree $deg_G(v)$ of v refers to $|N_G(v)|$, and $\Delta(G) = \max\{deg_G(v)\colon v\epsilon V(G)\}$. The closed neighbourhood of v is the set $N_G[v]$ = $N_G(v)\cup$ v for S$\subseteq V(G)$, $N_G(S) = \cup_{v\epsilon s} N_G(v)$ and $N_G[v]$= $N_G(S)\cup S$. If $N_G[v]$==V(G), then S is a dominating set in G. The minimum cardinality among dominating sets in G is called the *domination number* of G and is denoted by $\gamma(G)$.

Definition :If T is a regular of degree 2,every component is a cycle and regular graphs of degree 3 are called cubic.

Definition :If all the edges of the girth are the edges of any other cycles in a graph G.

Theorem :Let x be a line of a connected graph G, The following statements are equivalent(1)x is a bridge of G.(2) x is not on any cycle of G.(3)There exist points u and v of G s.t the line x is on every path joining u and v.(4)There exists a partition of v into subsets U and W s.t for any points u∈Uand w∈W the line x is on every path joining u and w.

Theorem:Let G be a connected graph with atleast three points. The following statements are equivalent. (1)G is a block (2)Every two points of G lie on a common cycle (3)Every point and line of G lie on a common cycle (4) Every two lines of G lie on a common cycle (5)Given two points and one line of G, there is a path joining the points which contains the line (6)For every three distinct points of G, There is a path joining any two of them which contains the third.

A set $x \in S$ is said to be redundant in S if $N[x] \subseteq N[S-\{x\}]$ or N[x]∩N[S-x]= $\emptyset$ otherwise x is said to be irredundant in S . Finally , S is called an irredundant set if all $x \in S$ are irredundant in S , Otherwise S is a redundant set .

The Corona G$\circ H$ of a graphs G and H is the graph obtained by taking one copy of G and $|V(G)|$ copies of H and then joining the i^{th} vertex of G to every vertex in the i^{th} copy of H . It is customary to denote by H_v that copy of H whose vertices are adjoined with the vertex v of G. In effect G$\circ H$ is composed of the subgraphs H_v+v joined together by the edges of G and its cartesian product of G and H. Moreover V(G$\circ H$) = $\cup_{v\epsilon V(G)} V(H_v + v)$ and let S$\subseteq V(G \circ H)$. If S is a dominating set in G$\circ H$ then $S\cap V(H_v + v) \neq \emptyset$, $\forall v\epsilon V(G)$

2.MAIN RESULTS

DEFINITION2.0: A SET D⊂V(G) is called a irredundant girth 2-dominating set of G if every vertex in V-D is adjacent to at least two vertex in the girth(cycle) graph of G. The minimum cardinality of a irredundant girth 2-dominating set of G is called irredundant girth 2-domination number of G∘ H denoted by γ_{m2irg}(G ∘ H) also the addition of any edge decreases the irredundant girth 2-domination number denoted by γ_{m2irg}(G ∘ H).

Example2.1: For any corona graph $|G \circ H|$=K_5 =$(C_{n-2} + 2v) - 2e = 5$ and $\cup N(v_i) = C_3$ has girth 2-dominating set of G with γ_{m2irg} $(G \circ H)$=n-2=3 for n= 5 if Max{d(u_i, u_j)} ≥n-3,$i \neq j$ where $u_i \in C_3$ and $v_i \in (V-D)$. Hence γ_{mirg} (G)=n-1 with $|M| = 2$ and N[x]∩N[D-x]≠ ∅ .Since $|N(u_1) \cap (V-D)| = |(u_2, u_3, v_1) \cap (v_1, v_2)| = 1$,i ≠ 1 ,$|N(u_2) \cap (V-D)| = |(u_1, u_3) \cap (v_1, v_2)| = \emptyset$,i ≠ 1 and $|N(u_3) \cap (V-D)| = |(u_1, u_2, v_2) \cap (v_1, v_2)| = 1$,i ≠ 1 but we have $|N(v_1) \cap (D)| = |(u_1, v_2) \cap ((u_1, u_2, u_3)| = 1$,i ≠ 1 but it should be equal to 2 , that is $|N(v_i) \cap (D)| = 2$,i ≠ 1 hence we must have $|M| = 2$.

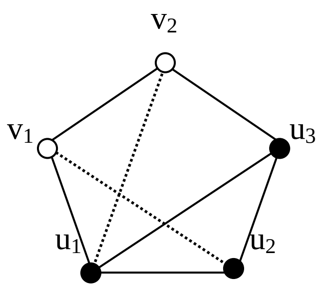

Figure 1: a Irredundant girth 2-dominating set γ_{m2irg}(G ∘ H)=n-2=3 with $|M| = 2$

Example 2.2: Every Corona graph of a girth graph G is G∘H = $C_4 \circ k_1$has a girth 2-dominating set if V-D=(G-D)∪ $H_v$and $|\cup N(H_v)| = C_4$.

Suppose if $\cup N(v_i) = C_4$ has a irredundant girth 2-dominating set of G with γ_{m2irg}(G ∘ H)=n=4 for n= 4 if Max{d(u_i, u_j)} ≥n-1,$i \neq j$ where $u_i \in C_4$. Hence γ_{m2irg}(G ∘ H)=n with $|M| = 4$.since$|N(u_i) \cap (V-D)| = 2$,i≠1 where $u_i \in S$ but we have $|N(v_1) \cap (D)| = |(u_1) \cap ((u_1, u_2, u_3, u_4)| = 1$,i ≠ 1 but it should be equal to 2 , that is $|N(v_i) \cap (S)| = 2$,i ≠ 1 hence we must have $|M| = 4$.

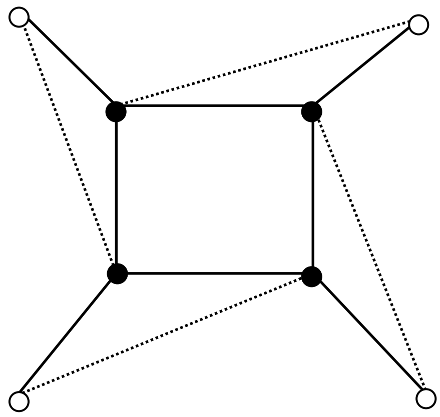

Figure 2: a Matching irredundant girth 2-dominating set $\gamma_{m2irg}(G \circ H)$=n=4 with $|M| = 4$

Hence $\gamma_{m2irg}(G \circ H)$= n with $|M| = 4$. But we have since$|N(u_i) \cap (V-D)| = 1$,i≠1 then we have $\gamma_{mr2g}(G \circ H)$= n with $|M| = 0$. Also we cannot have $\gamma_{m2irg}(G \circ H)$= n-1 since by theorem we have $|M|$ cannot be equal to 4 since by theorem every connected graph is of girth dominating set C_3 with $|M| \leq 3$.

Hence we must have $|\bigcup N(v_i)| = 4$. Hence if $|M| = 3$ then we can have atleast one v_i ∈V-D

and $\bigcup N(v_i) \neq C_4$ and if $|M| = 2$ then we have atleast 2 v_i ∈V-S and $\bigcup N(v_i) \neq C_4$ and if $|M| = 1$ then we have atleast 3 v_i ∈V-S and $\bigcup N(v_i) \neq C_4$. Hence we have $\bigcup N(v_i) = C_4$ and its $|N(v_i) \cap (D)| = 2$ also its Max{d(u_i, u_j)} = 3. Hence its $\gamma_{m2irg}(G)$=4 with $|M| = 4$.

Example 2.3: Every Corona graph of a girth graph G is G∘H = $C_3 \circ k_1$has a girth 2-dominating set if V-D=(G-D)∪ $H_v$and $|\bigcup N(H_v)| = C_3$is of matching irredundant girth 2-dominating set C_3 with $|M| \leq 3$.

Suppose if $\bigcup N(v_i) = C_3$ has a matching irredundant girth 2-dominating set of G with $\gamma_{m2irg}(G)$=n=3 for n= 3 if Max{d(u_i, u_j)} ≥n-1,$i \neq j$ where $u_i \in C_3$ N[x]∩N[D-x]≠ ∅. Hence $\gamma_{m2irg}(G)$=n with $|M| = 3$ since$|N(u_i) \cap (V-S)| = 2$,i≠1, where $u_i \in S$ but we have $|N(v_1) \cap (D)| = |N(v_1) \cap (u_1, u_2, u_3)| = 1$,i ≠ 1 but it should be equal to 2 , that is $|N(v_i) \cap (D)| = 2$,i ≠ 1 hence we must have $|M| = 3$. Hence $\gamma_{m2irg}(G \circ H)$= n with $|M| = 3$. But we have since$|N(u_i) \cap (V-D)| = 1$,i≠1 then we have $\gamma_{mir2g}(G \circ$

H)= n with $|M| = 0$. Also we cannot have γ_{m2irg}(G ∘ H)= n-1 since by theorem we have $u_i \in C_3$ and by theorem every connected graph is of matching irredundant girth 2-dominating set C_3 with $|M| \leq 3$.

Example2.4: Every Corona graph of a girth graph G is G∘H = $(C_4 + e) \circ k_1$has a girth 2-dominating set if V-D=(G-D)∪ $H_v$and $|\cup N(H_v)| = C_3$is of girth 2-dominating set C_3 with $|M| \leq 3$. Suppose if $\cup N(v_i) = C_3$ and N[x]∩N[D-x]≠ ∅has a matching irredundant girth 2-dominating set of G with $\gamma_{m2irg}(G)$=n-1=3 for n= 4 if Max{d(u_i, u_j)} ≥n-2,$i \neq j$ where $u_i \in C_3$. Hence γ_{m2irg}(G ∘ H)=n-1 with $|M| = 3$ since$|N(u_i) \cap (V - D)| = 2$,i≠1, where $u_i \in S$ but we have $|N(v_1) \cap (D)| = |N(v_1) \cap (u_1, u_2, u_3)| = 1$, i ≠ 1 but it should be equal to 2 , that is $|N(v_i) \cap (D)| = 2$, i ≠ 1 hence we must have $|M| = 3$. Hence $\gamma_{m2irg}(D)$= n-1 with $|M| = 3$. Also we can have γ_{m2irg}(G ∘ H)= n-1 since by theorem we have $u_i \in C_3$ and by theorem every connected graph is of matching irredundant girth 2-dominating set C_3 with $|M| \leq 3$.

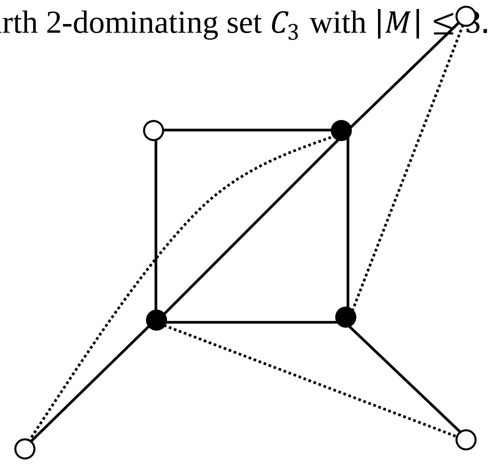

Figure 3: a Matching Irredundant girth 2-dominating set γ_{m2irg}(G ∘ H)=n-1 with $|M| = 3$

Lemma 2.4: Every Corona graph of a girth graph G is G∘H = $C_n \circ k_1$has a matching irredundant girth 2-dominating set if V-D=(G-D)∪ $H_v$and $|\cup N(H_v)| = C_n$ of G and N[x]∩N[D-x]≠ ∅with $\gamma_{m2irg}(G)$= n with $|M| = n$ for all n≥ 3 since where $u_i \in D$.

Proof: Every Corona graph of a girth graph G is G∘H = $C_n \circ k_1$has a matching irredundant girth 2-dominating set if V-D=(G-D)∪ $H_v$and $|\cup N(H_v)| = C_n$.

Suppose if $\cup N(v_i) = C_n$ has a matching irredundant girth 2-dominating set of G with $\gamma_{m2irg}(v)$=n for ≥ 3if Max{d(u_i, u_j)} ≥n-1,$i \neq j$ where $u_i \in C_n$ and N[x]∩N[D-x]≠ ∅. Hence $\gamma_{m2irg}(G)$=n with $|M| = n$.since$|N(v_i) \cap (D)| = 2$,i≠1 and since $|N(v_i) \cap (D)| = 2$,i≠1 where $u_i \in D$.Hence γ_{m2irg}(G ∘ H)= n with $|M| = n$. But we have since$|N(v_i) \cap (D)| = 1$,i≠1 then we have γ_{mrg} (G $\circ H$)= n with $|M| = 0$. Also we cannot have γ_{m2irg} (G $\circ H$)= n-1 since by theorem we have $u_i \in C_n$ since by theorem every connected graph is of irredundant girth dominating set C_n with $|M| \leq n$.

Suppose if $\cup N(v_i) = C_{n-1}$ not having a irredundant girth 2-dominating set of G with $\gamma_{m2g}(G)$=n-1 for ≥ 4 if Max{d(u_i, u_j)} ≥n-2,$i \neq j$ where $u_i \in C_{n-1}$ and N[x]∩N[D-x]≠ ∅ but atleast one$u_i \notin C_n$=D hence $|N(v_i) \cap (D)| \neq 2$ and its $\gamma_{m2g}(\mathrm{G} \circ H) \neq n-1$ but $|M|$ cannot be equal to n-1 since by theorem every connected graph is of girth 2-dominating set C_n with $|M| \leq n$.Hence we have $|\cup N(H_v)| = C_n$ of G with $\gamma_{m2irg}(G)$= n with $|M| = n$ for all n≥ 3.

Lemma 2.6:ACororna graph G∘H has a matching irredundant girth 2 dominating set if the addition of any edge decreases the matching irredundant girth 2-domination number and increases to$|N(v_i) \cap (D)| = 2$,i $\neq 1$

Proof : Suppose G is any graph G∘H_v is any corona graph of G. Suppose C_n is the girth graph of G and N[x]∩N[D-x]≠ ∅its Max{d(u_i, u_j)} $= n-1$ hence its $\gamma_{m2irg}(G)$=n with $|M| \leq n$ with $|N(v_i) \cap (D)| = 2$ and its $|\cup N(v_i)| = n$.

If $\cup N(v_i)$= C_{n-1} which implies that there will be Max{d(u_i, u_j)} $= n-1$ and N[x]∩N[D-x]≠ ∅$|\cup N(v_i)|$ ≠2 hence we must have $|N(v_i) \cap (D)| = 2$,i $\neq 1$ it is a matching irredundant girth 2-dominating set of G.

If $|N(v_i) \cap (D)| \neq 2$ hence we must have $|\cup N(v_i)| = 2$.To find $|\cup N(v_i)| = 2$, If $|V-D|$=$|D|$ then we can have a matching M for all S. Hence we have N[x]∩N[D-x]≠ ∅, $|N(v_i) \cap (D)| = 2$,i ≠ 1 and we have $\gamma_{m2irg}(G)$=n with $|M| \leq n$.

Theorem 2.11: For every Corona graph of a girth graph G is G∘H = $C_6 \circ k_1$has a matching irredundant girth 2-dominating set if V-D=(G-D)∪ H_v , $\mathrm{N[x]} \cap \mathrm{N[D-x]} \neq \emptyset$and $|\cup N(H_v)| = C_n$ of G with γ_{m2irg}(G ∘ H)= n with $|M| = n$ for all n≥ 3 since where $u_i \in D$.

Proof: Every Corona graph of a girth graph G is G∘H = $C_n \circ k_1$has a matching irredundant girth 2-dominating set if V-D=(G-D)∪ $H_v m$$\mathrm{N[x]} \cap \mathrm{N[D-x]} \neq \emptyset$and $|\cup N(H_v)| = C_n$.

Suppose if $\cup N(v_i) = C_n$ has a matching irredundant girth 2-dominating set of G with γ_{m2irg}(G ∘ H)=n for ≥ 3if Max{d(u_i, u_j)} ≥n-1,$i \neq j$ where $u_i \in C_n$. Hence γ_{m2irg}(G ∘ H)=n with $|M| = n$.since$|N(v_i) \cap (S)| = 2$,i≠1 where $u_i \in S$.

If Max{d(u_i, u_j)} = 2 and also $\cup N(v_i) = C_3$we have $C_{n-3} + 3v$=6 and 3v=6-C_{n-3} and V-S = 6x1 +3v =9 vertices are adjacent with $C_3$and 3x1+1=4 vertex is non adjacent with C_3 and 3x1+2v= 3+2=5 vertices are adjacent with C_3.Now we have $|N(u_i) \cap (V-D)| = 1$, i ≠ 1 but $|N(v_i) \cap (S)| \neq 2$,i≠1, N[x]∩N[D-x]≠ ∅ since we have $|M| \leq 3$ for all C_3 but 4 vertices are non adjacent hence we must have 4x2=8 vertex is adjacent since $|N(v_i) \cap (D)| = 2$ and 5x1 +8=13 matching needed to have $|N(v_i) \cap (S)| = 2$, N[x] ∩ N[D − x] ≠ ∅ since we have $|M| \leq 3$ for all C_3 hence we cannot have $|M| = 3$ for all C_3.

If Max{d(u_i, u_j)} = 3 and also $\cup N(v_i) = C_4$since $|N(v_i) \cap (D)| = 2$ and 8 +4=12 matching needed to have $|N(v_i) \cap (D)| = 2$, N[x] ∩ N[D − x] ≠ ∅ since we have $|M| = 4 \leq 8$ for all C_4 hence we cannot have $|M| = 4$ for all C_4.

If Max{d(u_i, u_j)} = 4 since we have $|M| = 5 \leq 7$ for all C_5 hence we cannot have $|M| = 5$ for all C_5. Hence G∘H = $[C_6 \circ k_1] - e$, N[x] ∩ N[D − x] ≠ ∅has a matching irredundant girth 2-dominating set and γ_{m2irg}(G ∘ H) = 5 with $|M| = 5$.

If Max{d(u_i, u_j)} = 5 since we have $|M| = 6 \leq 6$ for all C_6 hence we can have $|M| = 6$ for all C_6. Hence G∘H = $[C_6 \circ k_1]$ has a matching irredundant girth 2-dominating set , N[x]∩N[D-x]≠ ∅and $\gamma_{m2irg}(G \circ H) = 6$ with $|M| = 6$.

Theorem 2.16: Suppose for any graph $|G \circ$ H $|$=($C_n or(C_n + e) or(C_n + 2e)) \circ (k_{1,m})$ and $\cup N(v_i) = C_3$, N[x] ∩ N[D − x] ≠ ∅ has a matching irredundant girth 2-dominating set of G with $\gamma_{m2irg}(G \circ H)$= n with $|M| = n$ for all n≥ 3 since where $u_i \in S$.

Proof: Suppose if $\cup N(v_i) = C_n$ has a matching irredundant girth 2-dominating set of G with $\gamma_{m2irg}(G \circ H)$=n for n ≥ 3if Max{d($u_i, u_j$)} ≥n-1,$i \neq j$ where $u_i \in C_n$, N[x] ∩ N[D − x] ≠ ∅ . Hence $\gamma_{m2irg}(G \circ H)$=n with $|M| = n$.since$|N(v_i) \cap (D)| = 2$,i≠1 where $u_i \in D$.

If Max{d(u_i, u_j)} $= n - 1$ and also $\cup N(v_i) = C_n$we have $C_n + 0v$=6 and v=6-C_n and V-D = 2m vertices are adjacent with $C_n$and 0 vertex is non adjacent with C_n and 0x3+0v= 0 vertices are adjacent and 2xm vertices are adjacent with C_n.Now we have $|N(u_i) \cap (V-D)| = 1$, i ≠ 1 but $|N(v_i) \cap (D)| \neq 2$,i≠1 , N[x]∩N[D-x]≠ ∅ to make its matching irredundant girth 2 dominating set of G since we have $|M| \leq m$ for all $C_n$but 0 vertices are non adjacent hence we must have 2xm vertex is adjacent since and m matching needed to have $|N(v_i) \cap (D)| = 2$, $N[x] \cap N[D-x] \neq \emptyset$ since we have $2m \leq n$ for all C_n hence we can have $|M| = 2m$for all C_n with $\gamma_{m2irg}(G)$= n.

If $\text{Max}\{d(u_i, u_j)\} = n-2$ and also $\cup N(v_i) = C_{n-1}$we have $C_{n-1} + 1v$=n and v=n-$C_{n-1} = n-(n-1)$ and V-S = 2m+v=2m+1 vertices are adjacent with $C_{n-1}$and also either 1m+0m or 0m+0v=0 vertex is non adjacent with C_{n-1} and either 1m+1v=m+1 or 2m+1v=2m+1 vertices are adjacent with C_{n-1} since 2m edges incident on 2 vertices .Now we have $|N(u_i) \cap (V-D)| = 1$, $i \neq 1$ but $|N(v_i) \cap (D)| \neq 2$, $N[x] \cap N[D-x] \neq \emptyset$,i≠1 to make its matching irredundant girth 2 dominating set of G since we have $|M| \leq m$ for all hence we must have 2m matching needed to have $|N(v_i) \cap (S)| = 2, N[x] \cap N[D-x] \neq \emptyset$ since we have $2m \leq n-1$ for all C_{n-1} hence we can have $|M| = 2m$for all C_{n-1} with $\gamma_{m2irg}(G \circ H)$= n-1.

If $\text{Max}\{d(u_i, u_j)\} = n-3$ and also $\cup N(v_i) = C_{n-2}$we have $C_{n-2} + 2v$=n and v=n-$C_{n-2} = n-(n-2)$ and V-D = 2m+2v=2m+2 vertices are adjacent with $C_{n-1}$and also either 2m+0v=2m or 1m+1v=m+1 vertex is non adjacent with C_{n-2} and also either 0m+2v=2 or 1m+1v=m+1 or 2m+2v=2m+2 vertices are adjacent with C_{n-2} since 2mx2+2x1 or (m+1)x2 +(m+1) or 0x2+(2m+2)x1 matching needed to have $|N(v_i) \cap (D)| = 2, N[x] \cap N[D-x] \neq \emptyset$ since we have $2m \leq n-2$ for all C_{n-1} hence we can have $|M| = 2m$for all C_{n-1} with $\gamma_{m2irg}(G)$= n-1.

Theorem 2.18: Suppose for any corona graph $G \circ H \cong (C_4 + \cup_{i=1}^{4} H_i) - 4e$where H_i is a singleton vertex and $\cup N(v_i) = C_4$ has a matching irredundant girth 2-dominating set of G and N[x]∩N[D-x]≠ ∅with $\gamma_{m2irg}(G \circ H)$= 4 with $|M| = 0$ for all n≥ 4 since where $u_i \in D$.

Proof: Suppose if $\cup N(v_i) = C_4$ has a irredundant girth 2-dominating set of $G \circ H$ with $\gamma_{m2irg}(G \circ H)$=4 for $n \geq 4$if $\text{Max}\{d(u_i, u_j)\}$ ≥n-1,$i \neq j$ where $u_i \in C_n$, N[x]∩N[D-x]≠ ∅. Hence $\gamma_{m2irg}(G \circ H)$=4 with $|M| = 0$. since$|N(v_i) \cap (D)| = 2$,i≠1 where $u_i \in D$.

If $\text{Max}\{d(u_i, u_j)\} = 3$ and also $\cup N(v_i) = C_4$we have $C_4 + 4v$=8 and C_4=8-4 and C_4=4=D and V-D=$\cup_{i=1}^{4} H_i$=4, each H_i =K_1 and 4x1=4 non adjacent with C_4 by exactly one vertex of C_4 then we have $|N(u_i) \cap (V-D)| = 1$, $i \neq 1$ and $|N(v_i) \cap (D)| = 3$,i≠1 and N[x]∩N[D-x]≠ ∅also we have $|M| = 0$ with $\gamma_{m2irg}(GH)$=4

Suppose $|M| = 1$ then we have $|N(u_i) \cap (V - D)| = 1$, i ≠ 1 and $|N(v_i) \cap (D)| \geq 2$,i≠1 ND N[x]∩N[D-x]≠ $\emptyset$hence we have matching irredundant girth 2-dominating set of G∘ H with $|M| = 1$

Suppose for any corona graph G∘ $H \cong (C_4 + \cup_{i=1}^{4} H_i) - 8e$ where H_i is a singleton vertex and $\cup N(v_i) = C_4$ has a matching irredundant girth 2-dominating set of G with $\gamma_{mir\,2g}(G \circ H)$= 4 with $|M| = 0$ for all n≥ 4 where $u_i \in D$.

If Max{d(u_i, u_j)} = 3 and also $\cup N(v_i) = C_4$we have $C_4 + 4v$=8 and C_4=8-4 and C_4=4=D and V-D =$\cup_{i=1}^{4} H_i$=4, each H_i =K_1 and 4x1=4 non adjacent with C_4 by exactly one vertex of C_4 then we have $|N(u_i) \cap (V - D)| \geq 1$, i ≠ 1, N[x] ∩ N[D − x] ≠ $\emptyset$ and $|N(v_i) \cap (D)| = 2$,i≠1 also we have $|M| = 0$ with $\gamma_{m2irg}(G \circ H)$= 4.

Suppose $|M| = 1$ then we have Max{d(u_i, u_j)} = 2 also $|N(u_i) \cap (V - D)| = 1$, i ≠ 1, N[x] ∩ N[D − x] ≠ $\emptyset$ and $|N(v_i) \cap (D)| = 2$,i≠1 hence we have a matching irredundant girth 2-dominating set of G∘ H with $|M| = 1$ and its $\gamma_{m2irg}(G \circ H)$= 3

Suppose for any corona graph G∘ $H \cong (C_4 + \cup_{i=1}^{4} H_i) - 12e$ where H_i is a singleton vertex and $\cup N(v_i) = C_4$ has a matching irredundant girth 2-dominating set of G with $\gamma_{m2irg}(G \circ H)$= 4 with $|M| = 4$ for all n≥ 4 where $u_i \in D$.Since we have Max{d(u_i, u_j)} = 3 and $|\cup N(v_i)| = 4$ also $|N(u_i) \cap (V - D)| = 1$, i ≠ 1, N[x] ∩ N[D − x] ≠ ∅ and $|N(v_i) \cap (D)| = 1$,i≠1 hence we have 4 matching needed to have $|N(v_i) \cap (D)| = 2$,i≠1 .

If $|M|$ =5 , then we have Max{d(u_i, u_j)} = 2 and $|\cup N(v_i)| = 4$ also $|N(u_i) \cap (V - D)| = 1$, i ≠ 1, N[x] ∩ N[D − x] ≠ ∅ and $|N(v_i) \cap (D)| = 1$,i≠1 hence we have 5 matching needed to have $|N(v_i) \cap (D)| = 2$,i≠1 . Now we have a $\cup N(v_i) = C_4$ has a matching irredundant girth 2-dominating set of G∘ H with $\gamma_{m2irg}(G \circ H)$= 3 since we have $|M| \leq 4$ for all C_4 hence we cannot have $|M| = 5$ for all C_4.

Suppose for any corona graph G∘ $H \cong (C_n + \cup_{i=1}^{n} H_i) - ne$ where H_i is a singleton vertex where n= $C_n + n = 2n$,$C_n = 2n - n$,$C_n = n$ and $\cup N(v_i) = C_n$ and V-S=$\cup_{i=1}^{n} H_i$ =n and nx1=n vertices are non adjacent with C_n by exactly one vertex in D has a matching irredundant girth 2-dominating set of G with $\gamma_{m2irg}(G \circ H)$= n with $|M| = 0$ for all n≥ 4 where $u_i \in D$.Since we have Max{d(u_i, u_j)} $= n - 1$ and $|\cup N(v_i)| = n$ also $|N(u_i) \cap (V - D)| = 1$, i ≠ 1, N[x] ∩ N[D − x] ≠ ∅ and $|N(v_i) \cap (D)| = n - 1$,i≠1.

Suppose for any corona graph G∘ H $\cong (C_n + \cup_{i=1}^{n} H_i) - 2ne$ where H_i is a singleton vertex where n= $C_n + n = 2n$,$C_n = 2n - n$,$C_n = n$ and $\cup N(v_i) = C_n$ and V-D=$\cup_{i=1}^{n} H_i$

=n and nx1=n vertices are non adjacent with C_n by exactly two vertex in S has a matching irredundant girth 2-dominating set of G with $\gamma_{m2irg}(G \circ H)$= n with $|M| = 0$ for all n≥ 4 where $u_i \in S$.Since we have Max{d(u_i, u_j)} $= n - 1$ and $|\cup N(v_i)| = n$ also $|N(u_i) \cap (V - D)| = 1$, i ≠ 1, N[x] ∩ N[D − x] ≠ ∅ and $|N(v_i) \cap (D)| = n - 2$,i≠1.

Suppose for any corona graph G∘ $H \cong (C_n + \cup_{i=1}^{n} H_i) - 3ne$ where H_i is a singleton vertex where n= $C_n + n = 2n$,$C_n = 2n - n$,$C_n = n$ and $\cup N(v_i) = C_n$ and V-D=$\cup_{i=1}^{n} H_i$ =n and nx1=n vertices are non adjacent with C_n by exactly three vertex in S has a matching irredundant girth 2-dominating set of G with $\gamma_{m2irg}(G \circ H)$= n with $|M| = 0$ for all n≥ 4 where $u_i \in D$.Since we have Max{d(u_i, u_j)} $= n - 1$ and $|\cup N(v_i)| = n$ also $|N(u_i) \cap (V - S)| = 1$, i ≠ 1 , N[x] ∩ N[D − x] ≠ ∅ and $|N(v_i) \cap (D)| = n - 3$,i≠1.

If $|M| = n$ then we have Max{d(u_i, u_j)} $= n - 2$ and $|\cup N(v_i)| = n$ also $|N(u_i) \cap (V - D)| = 1$, i ≠ 1 and $|N(v_i) \cap (D)| = n - 3$,i≠1 hence we have $\gamma_{m2irg}(G)$= n with $|M| = n$.

If $|M| = n + 1$ then we have Max{d(u_i, u_j)} $= n - 2$ and $|\cup N(v_i)| = n$ also N[x] ∩N[D-x]≠ ∅, $|N(u_i) \cap (V - D)| = 1$, i ≠ 1 and $|N(v_i) \cap (D)| = 1$,i≠1 hence we have $\gamma_{m2irg}(G \circ H)$= n-1 since we have $|M| \leq n$ for all C_n hence we cannot have $|M| = n + 1$ for all C_n.

3.Conclusion :

In this paper we found an upper bound for the irredundant girth 2 domination number and Relationships between Matching irredundant girth 2 domination number and characterized the corresponding extremal graphs. Similarly also the addition of any edge decreases the irredundant girth 2 domination number denoted by $\gamma_{2mir\,2g}(G \circ H)$ with other graph theoretical parameters can be considered.

References

[1] R.B. Allan and R. Laskar, *On domination and independent domination numbers of a graph*, Discrete Mathematics, Vol. 23, No. 2, 73-76, 1978.

[

2] I.S. Aniversario, F.P. Jamil and S.R. Canoy Jr., *The closed geodetic numbers of graphs*, Utilitas Mathematica, Vol. 74, pp. 3-18, 2007.

[3] C. Berge, *theory of Graphs and its Applications*, Methuen, London, 1962.

[4] F. Buckley, F. Harary. *Distance in graphs*. Redwood City. CA: Addition-Wesley. 1990.

[5] W. Duckworth and N. C. Wormald, *On the independent domination number of random regular graphs*, Combinatorics, Probabilty and Computing, Vol. 15, 4, 2006.

[6] T. Haynes, S. Hedetniemi and M. Henning ,*Domination in graphs applied to electrical power networks*, J. Discrete Math. 15(4), 519-529, 2000.

[7] T.W. Hanes, S.T. Hedetniemi and P.J. Slater, *Fundamentals of Domination in Graphs*. Marcel Dekker, Inc. New York (1998).

[8] O. Ore, *Theory of graphs* , Amer. Math. Soc. Colloq. Publ., Vol.38, Providence, 1962.

[9] L.Sun and J. Wang, *An upper bound for the independent domination number*, Journal of Combinatorial Theory, Vol.76, 2. 240-246, 1999.

[10] H. Walikar, B. Acharya and E. Sampathkumar, *Recent developments in the theory of domination in graphs*, Allahabad, 1, 1979.

[11] Teresa L. Tacbobo and Ferdinand P.Jamil, *Closed Domination in Graphs*, International Mathematical Forum, Vol. 7, 2012, No. 51, 2509-2518.

[12] T.R.NirmalaVasantha, *A study on Restrained Domination number of a graph*, Thesis 2007, M.S.U, Tirunelveli.

[13] B.D.Acharya, H.B.Walikar, and E.Sampathkumar, *Recent developments in the theory of domination in graphs*. In MRI Lecture Notes in Math. Mehta Research Instit, Allahabad No.1, (1979).

[14] A.NellaiMurugan and G.Victor Emmanuel,*Degree Equitable Domination Number and Independent Domination Number of a Graph ,International Journal of Innovative Research in Science, Engineering and Technology(An ISO 3297:2007 Certified organization), Vol.2,Issue 11, November 2013.*

[15] A.NellaiMurugan and G.Victor Emmanuel, *Complete Dominating Number Of Graphs, Indian Journal Of Applied Research, Volume : 4 , Issue : 1 , Jan 2014 , ISSN - 2249-555X.*

[16] J. A.Bondy and U. S. R. Murty, Graph Theory, Springer, 2008.

[17] C. Berge, Theory of Graphs and its Applications, Methuen, London, 1962

[18] F .Harary, Graph Theory, Addison-Wisley Publishing Company, Inc

[19]A.NellaiMurugan, G.Victor Emmanuel, Triple Domination Number and Its Connectivity of Complete Graphs, Int. Journal of Engineering Research and Applications, ISSN : 2248-9622, Vol. 4, Issue 1(Version 9), January 2014, pp.07-11

[20]A.NellaiMurugan, G.Victor Emmanuel, Triple Domination Number and Its Chromatic Number of Graphs, Inter national Journal of Modern Sciences and Engineering Technology(IJMSET), Volume. 1, Issue 1,2014,pp.1-8.

[21]A.NellaiMurugan and Victor Emmanuel.G, *Restrained complete domination number of graphs,out reach journal ,VOC College* ,2015/Vol. No: VIII/Pg.No.146-154, ISSN(P):0975

1246 ISSN(O):2321 8835 Impact Factor 6.733

[22]NellaiMurugan.A and Victor Emmanuel.G,*Generalised Girth Domination Number of Graphs, International Journal of Mathematics and Computer research,Volume 4 ,issue 06 ,June 2016,PageNo.1404-1409, ISSN : 2320-7167*

[23]NellaiMurugan.A, Victor Emmanuel.G,*Irredundant Complete Domination Number of Graphs, International Journal of Emerging Trends in Science and Technology,Impact Factor: 2.838 , IJETST- Volume:03,Issue:03,Pages 3638-3646,March 2016,ISSN: 2348-9480*

[24] NellaiMurugan.A and G.VictorEmmanuel.G, International Journal of Creative research Thought, Restrained Girth Domination Number of Graphs, December 2017/Volume 5, Issue 4, ISSN: 2320-2882

[25] Beno.I,VictorEmmanuel.G, International Journal of Scientific & Technology Research , Center Location Problem in Graph Theory, Volume 9, Issue 03, Morch 2020.

[26] Shyamala.L,VictorEmmanuel.G,International Journal of Science and Research, On Matching Girth Domination Number of Graphs,Vol.8,Issue.12, Dec.2019,ISSN.2319-7064